Information Coefficient

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Market timing is simply a bet on the direction on the direction of the market. If a manager makes N bets on the direction of the market, N_C of these bets are correct, then the information coefficient of the manager is the covariance between the forecast ad the actual direction of the market: that is

$$IC = 2\left(\frac{N_C}{N}\right) - 1$$

Let's examine why.
Let's assume that market movement's direction can be encoded as

$$Y_i = \begin{cases} 1, & MKT\ UP \\ -1, & MKT\ DOWN \end{cases}$$

and portfolio manager's bets are represented as

$$X_i = \begin{cases} 1, & MKT\ UP \\ -1, & MKT\ DOWN \end{cases}$$

Based on the definition of the IC,

$$\begin{aligned} IC &= cov(X_i, Y_i) \\ &= \mathbb{E}[XY] - \mathbb{E}[X] \cdot \mathbb{E}[Y] \\ &= \frac{\sum_{i=1}^{N} X_i \cdot Y_i}{N} - \frac{\sum_{i=1}^{N} X_i}{N} \cdot \frac{\sum_{i=1}^{N} Y_i}{N} \end{aligned}$$

Note that

$$X_i \cdot Y_i = \begin{cases} 1, & X_i = Y_i \\ -1, & X_i \neq Y_i \end{cases}$$

If we assume that $\mathbb{E}[Y] = \frac{\sum_{i=1}^{N} Y_i}{N} = 0$, that is, there isn't a prior knowledge on how the market will move, then we can deduce that

$$\begin{aligned} IC &= \frac{\sum_{i=1}^{N} X_i \cdot Y_i}{N} \\ &= \frac{1 \cdot (\#of\ correct\ bets) - 1 \cdot (\#of\ incorrect\ bets)}{N} \\ &= \frac{N_C - (N - N_C)}{N} = 2\left(\frac{N_C}{N}\right) - 1 \end{aligned}$$

If the two information sources are correlated, the information coefficient (skill) of the manager does not increase proportionately with the amount of the new information. Rather, the combined information coefficient using the two sources of information

$$IC_{COM} = IC\sqrt{\frac{2}{1+r}}$$

where IC is the original information coefficient and r is the correlation between the two sources of the information.